Brane and Wormhole in Baby Universe

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Abstract

In this paper we search the minimal conditions for the creation of a Universe from a Tolman wormhole 'bounce' from a previous collapse without passing by an initial singularity. Inspired from brane cosmology, the total density is taken to be the sum of the ordinary matter and time-decreasing exotic matter, as well as time-decreasing cosmological constant. We show that these later didn't affect the standard point-wise energy conditions and that there is always an open region surrounding the bounce over which the strong energy condition must be violated. Flat and hyperbolic spatial Universes are not permitted in our model.

Recently there have been a number of studies and investigations concerning cyclic or oscillating Universes [1,2]. The inflationary model in which the Universe has undergone a period of exponential expansion, has successfully explained many problems in standard cosmology, but it didn't avoid the initial singularity problem [3]. The interest in cyclic or oscillating Universes had largely declined especially after the development of the first cosmological singularity problem [4,5]. But most of the cosmological studies concerning creation from a 'bounce' didn't take into account the presence of the cosmological constant or even curvature parameter or density perturbation. Recently Randall and Sundrum have proposed two models in which our universe is a three-brane imbedded in a five-dimensional anti-de-Sitter AdS_5 as a possible solution to the heirarchy problem between weak and Planck scales [6,7]. In contrast with the Kaluza-Klein approach, their models are based on the idea that standard model fields could be confined to a three-dimensional world, corresponding to our apparent Universe, while gravity belongs to a higher dimensional space. Two important questions arise concerning first the validity of these models with respect to the cosmological evolution of the Universe and second their agreements with recent observations. Due to the fact that the energy density of the brane is quadratic in the

brane Friedmann equations, under minimal conditions, the equations governing the cosmological evolution of the brane are different from those derived in standard cosmology [8]. In a recent paper [9], we shown show that the brane-Friedmann equations are identical to those derived in the standard 4D hot Big-Bang model but with an additional density term playing the role of the gravitino-density. In other words, we supposed that the total density is in fact the sum of the ordinary matter and exotic one in the form $\rho = \rho|_{matter} + \rho|_{exotic}$ where $\rho|_{exotic} = \frac{3m^2}{8\pi G}$, m being a constant having the dimensions of time^{-2}.
In this paper, we will take in consideration these two points: the presence of a time-decreasing cosmological constant and time-decreasing density perturbation simultaneously. Quite generally, the metric is of the FRW type [10]. this is described by the metric:

$$ds^2 = -dt^2 + R(t)^2 \left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right] \quad (1)$$

where k is the curvature parameter and R is a scale factor. The Friedmann dynamical equation (in the presence of a cosmological constant Λ) is then described by:

$$\frac{\dot{R}}{R^2} + \frac{\kappa}{R^2} = \frac{8\pi G\rho}{3} + \frac{\Lambda}{3} + m^2 \quad (2)$$

Today, the cosmological constant has the incredibly small value, $\Lambda < 10^{-46} GeV^4$, whereas generic inflation models require that Λ has a large value during the inflationary epoch [11]. This suggests that Λ is decreasing with time. We inspired also from inflationary models and suppose that the density fluctuations also decrease with time. We suggest that Λ and m^2 varies a inverse of the square of the scale factor. The laws we propose are $\Lambda = \frac{3\beta}{R^2}$ and $m^2 = \frac{\alpha}{R^2}$ where α and β are two positive parameters. For calculus convenience, we suppose that $3m^2 = 2\Lambda$ or $\alpha = 2\beta$. The dynamical equations reduces to:

$$\rho = \frac{3}{\chi}\left[\frac{\dot{R}^2}{R^2} + \frac{k'}{R^2}\right]$$

$$p = -\frac{1}{\chi}\left[2\frac{\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{k'}{R^2}\right] \quad (3)$$

where $k' = k - 3\beta$ is the modified curvature parameter. Some equations of interest are:

$$\rho + p = \frac{2}{\chi}\left[-\frac{d^2}{dt^2} lnR + \frac{k'}{R^2}\right]$$

$$\rho - p = \frac{2}{\chi}\left[\frac{1}{3R^2}\frac{d^2R^3}{dt^2} + 2\frac{k'}{R^2}\right]$$

$$\rho + 3p = -\frac{6}{\chi}\left[\frac{\ddot{R}}{R}\right] \tag{4}$$

$$3\gamma - 2\dot{R}^2 + 2R\ddot{R} + (3\gamma - 2)(1 - 3\beta) = 0$$

where $p = (\gamma - 1)\rho$ is the state equation. In fact, according to [12], the minimal conditions for the creation of a FRW Universe from a bounce where the NEC (null energy condition, $\rho + 3p \geq 0$), WEC (weak energy condition, $\rho \geq 0, \rho + p \geq 0$), DEC (dominant energy condition, $\rho + 3p \geq 0, \rho \pm p \geq 0$) is violated are $k' = +1$ and $R\ddot{R} \leq 1$ According to [10], the energy conditions are minimized by taking the Universe to be hyperspherical ($k' = +1$) and by making the bounce sufficiently gentle ($R\ddot{R} \leq 1$). In our model, this implies $k - 1 = 3\beta$. It is easy to check that in the matter-dominated and the radiation-dominated ephochs, we need to have $\alpha = \beta = 0$;in other terms, the cosmological constant and the density perturbation play no role. But is it the case for a FRW toy model? For an identical toy model traversable wormhole, the metric for $k' = +1$ is [14]:

$$ds^2 = -dt^2 + (R_0^2 + \delta^2 t^2)\left[\frac{dr^2}{1 - r^2} + r^2(d\theta^2 + sin^2\theta d\phi^2)\right] \tag{5}$$

Explicit calculations of the stress-energy components yields:

$$\begin{aligned}
\rho &= \frac{3}{\chi}\left[\frac{(k - 3\beta)R_0^2 + \delta^2(k - 3\beta + \delta^2)t^2}{(R_0^2 + \delta^2 t^2)^2}\right] \\
p &= -\frac{1}{\chi}\left[\frac{(1 + 2\delta^2)(k - 3\beta)R_0^2 + \delta^2(k - 3\beta + \delta^2)t^2}{(R_0^2 + \delta^2 t^2)^2}\right] \\
\rho + p &= \frac{2}{\chi}\left[\frac{(k - 3\beta)(1 - \delta^2)R_0^2 + \delta^2(k - 3\beta + \delta^2)t^2}{(R_0^2 + \delta^2 t^2)^2}\right] \\
\rho - p &= \frac{2}{\chi}\left[\frac{(k - 3\beta)(2 + \delta^2)R_0^2 + \delta^2(k - 3\beta + \delta^2)t^2}{(R_0^2 + \delta^2 t^2)^2}\right] \\
\rho + 3p &= -\frac{6}{\chi}\left[\frac{(k - 3\beta)(\delta^2)R_0^2}{(R_0^2 + \delta^2 t^2)^2}\right]
\end{aligned} \tag{6}$$

The subscript 0 denotes quantities evaluated at the bounce $t = 0$. The pressure is not positive in the Morris-Thorne toy model, but this is not the case in our model. It is easy to check that for $k = 0$ or -1. The pressure could be positive even at $t = 0$. To keep $\rho + p > 0$, we need to have ($\delta < 1, k - 3\beta > 0$) or ($\delta > 1, k - 3\beta < 0$). For the first case, the only satisfied condition is ($k = +1, \beta < \frac{1}{3}$) and for the second case ($k = +1, \beta > \frac{1}{3}$). In these two cases, NEC is satisfied.

For $\delta < 1$ or $\delta > 1$, $\rho + 3p > 0$ provided that $k - 3\beta < 0$.

In summary, for $\delta < 1$ and $k - 3\beta > 0$, $(k = +1, \beta < \frac{1}{3})$, the NEC, WEC and DEC are all satisfied but violates the SEC. This implies that Λ is less than that predicted by the inflationary cosmology. The density perturbation and the decreasing cosmological constant didn't affect the standard point-wise energy condition for the tore geometry. Generalization of all these results to space-times more general than the FRW Universes will be done in subsequent paper.

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